

## Tom Fuller: a memoir

Tom Fuller was a master craftsman and distinguished scholar in the field of analytical mechanics, as applied to automatic control. He made major contributions over a range of topics which included relay control, the use of the Fokker-Planck equation for stochastic systems, the properties of algebraic characteristic equations, the history of automatic control, and many others. His work was characterized by great thoroughness and a meticulous attention to detail in the unravelling of the complexities of dynamical and algebraic systems. Tom would have excelled in the golden age of analytical mechanics, or in the Victorian heyday of applied dynamics. He had a high regard for minor Victorian luminaries like Routh. When the International Journal of Control was planning a special edition to mark Routh's centenary, we went to Peterhouse to



look at Routh's portrait, and I remember thinking how well they would have got on together. After retirement, he suddenly became an enthusiastic user of computers, although of the smaller, more personal and more controllable, variety. In qualitative terms the nature of his papers was unchanged, although the number of significant digits in his results increased greatly. His rigorously high standards were not attuned to the high-volume, more-vigour-than-rigour, standards of current academic life. He had at least two sabbaticals for the express purpose of writing a book, but returned after each to report a heightened and deepened appreciation of the problems involved.

I was delighted when he was made a Reader in Cambridge—somehow the appellation suited him perfectly. The library was his spiritual home, and time spent elsewhere had to be very solidly justified. Although happy to travel to libraries, he seldom attended seminars or conferences. I once bumped into him on the London underground on my way to Heathrow. 'Going to the British Library, Tom?' I asked. 'Yes. Where are you off to?' 'Berkeley and Stanford,' I replied. 'Whatever for?' he asked. Such a trip was regarded by him with the same mild incredulity which I had for visits to the Bibliotheque Nationale to verify some of Hermite's references. Tom was neat, precise, and reserved with an almost Diracian economy in the use of words. He spoke with crisp precision and a slightly staccato manner, often lightly stamping one foot for emphasis. This, together with an erect bearing, gave him a faintly military air, which was belied however by a monkish tonsure and an unfailing courtesy. He had a formidable grasp of the science of mechanics in its Victorian splendour—Airy, Routh, Maxwell, Thomson and Tait were known in intimate detail in their lives as well as in their works. Although very reserved, it was I found possible to have lengthy and animated conversations with him if and only if the topic was analytical mechanics. I only once had the great pleasure of making him laugh out loud. I had been at an international conference on applied mechanics and had given an invited survey on developments in automatic control. I told Tom how, during the questioning afterwards, I had been harangued by a very eminent applied mathematician as to why control engineers seemed to have such a marked preference for Hamiltonian as opposed to Lagrangian fomulations. While I pondered how to

handle this—was it a joke?—the questioner became impatient and gave his own answer. It was, he supposed, because we must think that the equations looked prettier. How Tom and I laughed—one of the world's best known applied mathematicians revealing in public that he didn't know the difference between the generalized particle and the wavefront descriptions of dynamical systems.

We had a long and fruitful relationship as colleagues in the Control Group of Cambridge University Engineering Department. I took over from him as Editor of the *International Journal of Control*, which he had effectively founded, and for which he became Consulting Editor. He was the ideal Editor: meticulous, scrupulous, scholarly and organized. He felt that it was taking up too much of his research time, and so he effortlessly metamorphosed into the journal's most prolific contributor. I found him unfailingly helpful when consulted on the field in which he was such an outstanding authority.

We only interacted socially when the rhythms of professional life were broken by the contingencies of illness. As a result, the only informal, social contacts which we had tended to be in hospitals. I visited him in Addenbrooke's when he was recovering from a heart attack, finding his bed without difficulty as it was the only one with a great pile of books beside it. We had many enjoyable conversations about analytical mechanics and its history. He repaid a social debt by coming to see me in the Evelyn Nursing Home when I was recovering from a major operation. I was far too unwell to talk, lying there connected to drip tubes and monitoring instruments, and feeling dreadful. There was a long, unhappy silence as Tom strove to find something appropriate to say. He suddenly brightened, perhaps remembering our shared interest in the history of science. He leaned forward excitedly, tapping the sheets. 'Alistair', he said, 'Did you know that this is the nursing home where Rutherford died?' I did, but had been trying hard not to think about it.

The world changes, and not always for the better. The kind of academic world in which a highly specialized, idiosyncratic researcher and scholar such as Tom could flourish is fast vanishing, and much that is valuable is being lost with it. Tom was *sui generis*, an indefatigable student in one of science's oldest traditions. Our subject is richer for his work, and we are all poorer by his death. I mourn his passing, as will all his former colleagues and research collaborators, and the very many vicarious colleagues who knew him only through the meticulous scholarship of his very many fine papers.

In piam memoriam, Alistair MacFarlane

# A. T. Fuller—biographical notes<sup>1</sup>

Anthony Thomas Fuller was born on the Isle of Wight on 6th July 1924 and grew up in the New Forest close to Redlynch. He attended Bishop Wordsworth's School in Salisbury and then went up to Peterhouse, Cambridge in 1942 for an abbreviated wartime degree course in Natural Sciences. After graduation in 1944 he joined the Ministry of Supply in Porton, Wiltshire for two years to develop automatic controls for stabilizing tank guns. Then followed several years' employment in industry, firstly with Henry Hughes Ltd as a research engineer on the design of pen-recording systems, and then with the geophysicist Oscar Weiss in Johannesburg working on instrument design for geophysical prospecting. After returning to England, he was employed by Waymouth Gauges and Instruments Ltd on the design of automatic controls for jet engines. By this time, he was feeling a lack of fulfilment in doing applied work in industry, and in 1954 he contacted Cambridge about the possibility of returning to do research.

At this time, the Control Group in Cambridge was rapidly expanding under the leadership of John Coales (who sadly also died last year on the 6th of June). Tom Fuller was duly engaged as a research assistant, and from this time until his retirement in 1983 he remained with the Department of Engineering at Cambridge. It is a measure of his dedication to scientific research that he accepted this position given the financial sacrifice that it would entail having a family and young children. In 1952 he had married Barbara Elmslie, and by the time he submitted his PhD in 1959 they had four children.

During his first year back at Cambridge he assisted the group with electronic developments and designed a radioactive noise generator for the analysis of stochastic processes (13, pp. 194–197) which was used by members of the group for several years. He decided he would like to attempt a PhD degree and was registered as a research student in 1955. He had already published several papers by this time, including work on aperiodicity, but considered the problem of optimal control of systems with saturation for his PhD work. The thesis was concerned with both deterministic and stochastic systems and worked out explicit formulae for the switching surfaces in low order examples.

A Senior Assistantship in Research in 1958 was followed by appointment to the post of Assistant Director of Research in 1961, the latter becoming a permanent appointment in 1971. He was promoted to Reader in

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Control Engineering in 1979 in recognition of his contributions to the control field. He also held a position as Senior Research Fellow at Churchill College from 1967–76. None of these posts had a requirement for undergraduate teaching, which he did not enjoy. He was never tempted by the offer of professorships, which would have involved many administrative duties. Money was of little interest to him.

The early work on optimal control which attracted much attention was the derivation of the optimal switching curves for the double-integrator plant with saturable input, with various integral performance criteria (13, 19, 25). For an integral quadratic criterion this is now called the Fuller problem. A discovery was made which became known as Fuller's phenomenon: that an optimal control which brings an initial state to the origin in finite time may require an infinite number of control switches. Eventually it was realised that this behaviour is far from exceptional, as shown in 'The Ubiquity of Fuller's Phenomenon' by I. A. K. Kupka, in Nonlinear Controllability and Optimal Control (ed. Sussmann), M. Dekker, 1990. For the Fuller problem, and in later studies involving the triple integrator (32), it was further observed that some optimum trajectories have the property that the ratio of any two successive switching intervals is constant. The occurrence of such trajectories was shown to lead to poor sensitivity properties when plants were subjected to parameter variations (48, 66). He returned to this topic of 'constant ratio trajectories' several times in his career, culminating in the comprehensive treatment in (97). The Fuller problem continues to be a great influence on the optimal control field, as is amply illustrated in the book of M. I. Zelikin and V. F. Borisov devoted to this subject (Theory of Chattering Control, Birkhäuser, 1994).

Explicit computations of optimal control laws for specific plants were a feature of many of Tom Fuller's papers on optimal control. It seems that he had several different motivations for such work. Firstly, closed-form formulae were useful as test examples for simulation techniques. The case of bounded-input control of (i) multiple integrators (32) or (ii) linear plants with lags having time-constants in integer ratio (54) are such instances. Secondly, such calculations could suggest alternative control laws which might not be truly optimal but nevertheless could give satisfactory performance, these being called sub-optimal controllers. For example, in (45, 47) a property of 'invariance' under changes of time and signal scale was established for some optimal control laws, which was then imposed on specific feedback laws (e.g. a linear combination of functions of single state-variables) to yield sub-optimal controllers. Thirdly, closed form formulae for low-order examples were a way to gain insight into the control of plants where a general theory is challenging, e.g. saturation limits plus time delays (39), relay plus stochastic noise (20, 75), min-max performance criteria for saturating plants (19, 71).

The simplest candidates for sub-optimal controllers for relay and saturating control systems are linear state feedbacks. He showed in (37) that such feedbacks could often given near-optimum performance for an integral-square criterion. However, in (43) he proved that a chain of integrators preceded by relay or saturation cannot be globally stabilized by linear state feedback, if the integrators number three or more. This result has been independently rediscovered in the course of recent work by H. J. Sussmann, E. D. Sontag, Y. Yang (*IEEE Transactions on Automatic Control*, 1994, pp. 2411–2425) and others to construct globally stabilizing controllers for linear systems subject to bounded controls.

Stochastic systems, and especially their behaviour and optimization in the presence of nonlinearities, were a lifelong interest. The well-known paper on the Fokker-Planck equation for the probability density in state space (41) confirmed an interest in explicit solutions which could serve as test cases for studies of computational methods. A systematic study of approximate techniques of stochastic bang-bang control (75-77) made use of an exact analysis of first and second order systems with relay and white noise; deficiencies in dynamic programming applied to discretized models were exposed in a similar way in (60, 81). His only paper on linear control design (62) illustrated a concern to bridge the gap between control theory and practice. Exploiting Wiener factorization techniques to solve a stochastic optimal control problem, he argued that it was usually essential to take account of all three of: (a) low-frequency disturbances, (b) measurement noise, and (c) control power constraints, to obtain realistic designs. In comparing the resulting controllers with classical ones, the Fuller trademark is again in evidence of closed-form expressions for optimal controllers in low-order cases.

Research on aperiodicity was carried out while he was still employed at Waymouth Gauges and Instruments Ltd (4), motivated in part by problems of avoiding resonance in feedback systems—a polynominal is said to be *aperiodic* if all its roots are real and simple. Independently of work by Meerov in the Soviet Union, he derived necessary and sufficient conditions for aperiodicity in terms of 'inner' determinants of a resultant matrix (see (72) for historical notes), and made observations on certain redundancies in the system of inequalities for both aperiodicity and Hurwitz stability in (6). He also independently discovered the stability criterion of Liénard-Chipart (9) and found a simplified proof. For the aperiodicity criterion, he showed that the only redundancies were for the degree three case, though he clearly suspected there might be other, more economical criteria in higher order cases (72, 92). The work on

stability criteria was concerned with the 'root-pair-sum' (not the tabular) method of Routh. Some redundant and some non-redundant criteria were identified (69, 90, 91), though this line of work has not yet produced a definitive 'minimal' set of inequalities which could rival the Liénard-Chipart criterion. For the corresponding matrix problem, making use of Kronecker sums and root-pair-sum ideas, he obtained (40) inequalities expressed in terms of the elements of the matrix for the eigenvalues to have negative real parts, and had some partial success in reducing redundancy—work which has been exploited in computing stability radii for uncertain A-matrices. Despite a large body of classical literature, he realised that the subject of stability criteria (for polynomials to have left half-plane roots) was not complete. His influence will be felt in any future search for the 'best' and least redundant set of inequalities.

By no means least among the contributions of Tom Fuller are the works on the history of control. A series of investigative papers have explored the background which led up to the famous 1868 paper of Maxwell on governors (63-64, 73, 82-85, 87-89, 101-102). In addition to the scholarly publication of manuscripts, letters etc. there are fascinating historical and technical commentaries, e.g. a suggested completion of an embryonic 'stochastic control' analysis of the stability of Saturn's rings (85), description of the mechanical background and development of governors (64, 82, 89, 101–102), history of linearization in theoretical mechanics and astronomy (83), notes on the emergence of stability criteria (64, 87), as well as the elucidation of a number of enigmatic points in Maxwell's paper. Undoubtedly, these works will have a lasting impact on our understanding of the early development of control science. Great service was also done in making the works of two other pioneers of control more accessible. The Routh volume (59) reprinted the original Adams prize essay of 1877 with much interesting additional material, and (94-95) provided, for the first time, an English version of Lyapunov's memoir on stability in the Fuller translation of the French edition.

Tom Fuller displayed a unique and original perspective in research. He carefully thought out for himself what was important and pursued it independently of current trends or fashions. Who else would have attempted to establish *rigorously* the correctness of the first twelve decimal places of all possible ratios, in constant-ratio trajectories for multiple-integrator plants up to 16th order (97)? His marvellously clear and readable style, as well as unfailing attention to the historical literature, is always evident, whether this be a study of root computation (96, 103), a proof of time-optimality of predictive control strategies (47, 49–51, 78), or the calculation of the periods of pseudo-random number

generators (61). In (28) he asks: what is control? He felt that definitions that equated control with feedback were not sufficiently general, and proposed the following definition: 'A control system is a dynamic system which keeps some of its variables within certain bounds by means of the complexity of some part of the system'. He argued that this would include a mechanical governor for control of steam engines, but would exclude an extremely large flywheel for the same purpose. The same paper discussed why optimal control using dynamic programming or the maximum principle often missed important classical control solutions such as integral action or phase lead, emphasising the need for methods to deal quantitatively with plant variability in design.

In the control community he was well-known for his work with the *International Journal of Control*. He was a founder member of the Editorial Board when it separated from the *International Journal of Electronics and Control* in 1965 until 1992 and was Consultant Editor from 1972 until 1989. More than half of his publications appeared in the *International Journal of Control* or its forerunner. In one of his few incursions into 'worldly pursuits' he became a Director of the publishers Taylor & Francis from 1973 to 1981, being elected to the Board to advise on the company's engineering publications.

Tom Fuller was a very private man, but with a sense of humour. Former PhD students considered him to be an outstanding supervisor, devoted and kind, but strict, who insisted on precise accuracy in mathematical theory and computations. Among his close research colleagues he was greatly revered as a friend and mentor, for the clarity of his style and for his intellectual and moral integrity.

Tom Fuller's professional life was very important to him, although there were other things more important such as family life. He was very good with young children, making models, drawing pictures and creating a little secret world. He liked to walk and cycle at weekends with his family, play croquet or play the piano music of Chopin and Schumann. He was an incurable collector of books, spending much time in bookshops searching out old and out-of-print volumes.

In Cambridge in 1977 he suffered his first heart attack, recovered satisfactorily but suffered two further attacks during the following four years. He decided to take retirement from his Readership in 1983, on health grounds. He and his wife moved to Woodgreen, Hampshire in 1994, close to the place where he spent his youth in the New Forest. He was able to continue with his research after his retirement and kept up a frequent correspondence with colleagues. On 7th June 1999 Tom Fuller died suddenly at his home leaving a widow Barbara, four daughters and five grandchildren.

### Acknowledgments

I am grateful to Dr John F. Barrett for supplying the list of publications and index (prepared by A. T. Fuller in April 1999) and other useful information. I am also grateful to the following individuals for helpful comments on an earlier draft: M. Evans, B. Fuller, K. Glover, P. Harman, E. I. Jury, H. J. Sussmann, W. M. Wonham, A. S. I. Zinober.

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#### Publisher's appreciation

As publishers of *International Journal of Control*, Taylor & Francis note our deep appreciation of Tom Fuller's contribution both to engineering, scientific publishing, and Taylor & Francis. Dr Fuller served as a Director of Taylor & Francis Ltd from 1973 to 1981, advising on publications within the engineering sciences. Dr Fuller was a founding editorial board member of the *International Journal of Control*, serving as a Consultant Editor, and, as the bibliography attests, author of Taylor & Francis 'control' books, editor and translator of A. M. Lyapunov's *The General Problem of Stability of Motion*, and author finally of many significant papers published by the Journal. Echoing our motto *Alere Flammam*, he truly fed the flame of learning.